Electrodynamics
ISI B.Math
Midterm Exam : February 26, 2026

Total Marks: 60
Time : 3 hours
Answer all questions

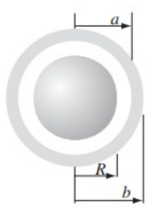
1. (**Marks: 2 + 6 + 7 = 15**)

(a) Write down the volume charge density ρ for the following charge configuration: A positive charge $+q$ located at $\mathbf{r} = \mathbf{a}$ and a negative charge $-q$ located at $\mathbf{r} = -\mathbf{a}$, where $\mathbf{a}$ is a constant vector.

(b) Suppose the electric field in some region is found to be $\mathbf{E} = kr^3\hat{\mathbf{r}}$ in spherical coordinates, where k is a constant. (i) Find the total charge contained in a sphere of radius R centred at the origin. (ii) Find the volume charge density $\rho(r)$.

(c) Find the potential inside and outside a uniformly charged solid sphere whose radius is R and whose total charge is q. Use infinity as your reference point. Sketch $V(r)$ roughly.

2. (**Marks: 6 + 2 + 7 = 15**)



(a)A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as in the figure above). The shell carries no net charge. (i) Find the surface charge density σ at R, at a, and at b. (ii) Find the potential at the center, using infinity as the reference point. (iii) Now the outer surface is touched to a grounding wire, which drains off charge and lowers its potential to zero (same as at infinity). How do your answers to (i) and (ii) change?

(b) It is given that $\nabla \cdot \mathbf{E} = C(\mathbf{r})$ and $\nabla \times \mathbf{E} = \mathbf{0}$. Is it possible to uniquely determine the electric field $\mathbf{E}$ from this data ? Explain.

(c) A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of the configuration.

3. (**Marks: 3 + 6 + 6 = 15**)

(a) Use the uniqueness theorem to show that the potential is constant inside an enclosure completely surrounded by conducting material, provided there is no charge within the enclosure.

(b) A conducting sphere of radius R has a charge Q_0 on it. A point charge q lies at a distance d from the centre of the sphere. Find the force between the point charge and the sphere using the method of images. Please make sure that you justify all the steps you use to arrive at this force.

(c) Two point charges $3q$ and $-q$ are separated by a distance a in two different configurations (i) $-q$ at (0,0,0) and $+3q$ at $(0, 0, a)$ (ii) $-q$ at $(0, 0, -a)$ $3q$ at $(0, 0, 0)$. Find the monopole moment and the dipole moment for each configuration. Find the approximate potential (in spherical coordinates) at large r (include both the monopole and dipole contribution).

4. (**Marks: 9 + 6**)

(a) The potential on the surface of a hollow sphere of radius R is given by $V_0(\theta) = k\sin^2\frac{\theta}{2}$. The general solution of the potential $V(r,\theta)$ arrived at by separation of variables in spherical polar coordinates is given by

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

Find the potential inside and outside the sphere and the surface charge density on the surface of the sphere.

(b) A sphere of radius R carries a polarization

$$\mathbf{P}(\mathbf{r}) = k\mathbf{r},$$

where k is a constant and $\mathbf{r}$ is the vector from the center.

(i) Calculate the bound charges σ_b and ρ_b.

(ii) Find the electric field $\mathbf{E}$ inside and outside the sphere.

Information you may or may not need:

$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\hat{\phi}$

$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$

$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$

$\int_{-1}^{1} P_l(x)P_m(x)\,dx = \frac{2}{2l+1}\delta_{lm}$

$P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2-1), P_3(x) = \frac{1}{2}(5x^3-3x)$